

# Real numbers

## Learn the idea. Show the method. Check your reasoning.

Three lessons and a final check. Use this pack alongside the interactive journey at [heliolearn.com/learn/grade-10/mathematics/real-numbers/](https://heliolearn.com/learn/grade-10/mathematics/real-numbers/).

### Your plan

Session 1: prime factors, about 15 minutes. Session 2: HCF and LCM, about 20 minutes. Session 3: irrationality, about 20 minutes. Allow another 10-20 minutes for checking and reflection. Take breaks as needed.

### Before you begin

Use a pencil and paper. You do not need to enter your name online. The solutions are in a separate teacher and answer guide. These are original Heliolearn questions, not a complete set of NCERT textbook solutions.

## Starting check

1. Which number is prime?

A. 1 B. 13 C. 21

2. Which number divides both 18 and 30?

A. 5 B. 9 C. 6

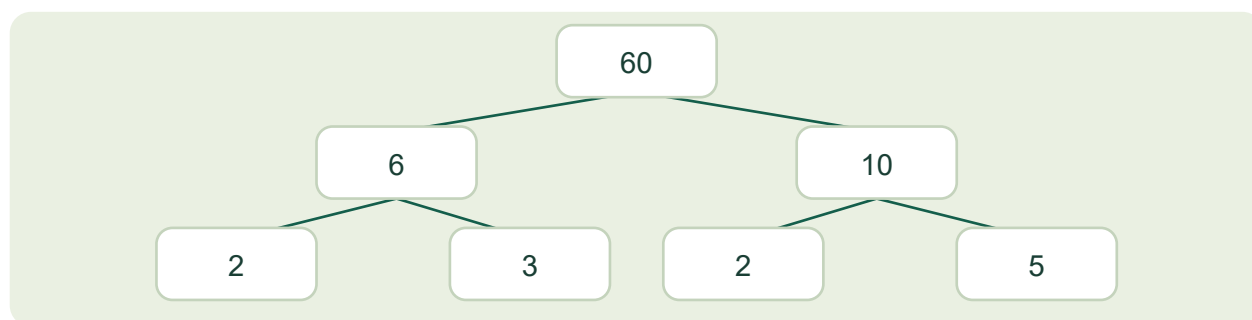
3. Which pair is in lowest terms?

A.  $\frac{6}{8}$  B.  $\frac{3}{4}$  C.  $\frac{9}{12}$

Circle a choice. Review a missed idea with a teacher or use the online recommendation. This starting check does not count toward completion.

# Build numbers from primes

Break a whole number into primes and explain why the final factors are unique.



## 1. Start with a product

Write 60 as  $6 \times 10$ . Both branches multiply back to 60, so the total has not changed.

## 2. Keep splitting composite factors

The number 6 becomes  $2 \times 3$ , and 10 becomes  $2 \times 5$ . Each remaining factor is prime, so stop splitting.

## 3. Collect repeated primes

The leaves are 2, 3, 2 and 5. Rearrange them:  $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$ . Multiplication order does not change the product.

## 4. Explain what stays the same

Starting with  $60 = 3 \times 20$  gives the same prime factors. For each whole number greater than 1, its prime-factor list is unique apart from order. This is the Fundamental Theorem of Arithmetic.

## Remember

Stopping at  $6 \times 10$  is a factorisation, but it is not a prime factorisation: 6 and 10 are composite.

## Try it and explain it

Choose an answer for each question. Use the space to show your working. A hint is available online if you need it.

1. Which is a prime factorisation of 72?

A.  $8 \times 9$  B.  $2 \times 2 \times 2 \times 3 \times 3$  C.  $2 \times 6 \times 6$

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2. Why is 1 not a prime number?

A. It has only one positive factor B. It is too small to multiply C. It is even

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3. Two correct factor trees for 90 must finish with...

A. the same branch shapes B. different prime factors C. the same primes, including repetitions

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### Explain in your own words

A friend writes  $90 = 9 \times 10$  and says the job is finished. Explain what must still be split and why.

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# Choose HCF or LCM

Use prime factors to solve equal-group and repeating-event problems.

$$18 = 2 \times 3 \times 3$$

$$30 = 2 \times 3 \times 5$$

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 2 \times 3 \times 3 \times 5 = 90$$

$$\text{Check: } 6 \times 90 = 18 \times 30 = 540$$

## 1. Factor both numbers

For 18 and 30:  $18 = 2 \times 3 \times 3$ ;  $30 = 2 \times 3 \times 5$ . Keep repeated factors because they affect the result.

## 2. Use the shared primes for HCF

Both lists contain one 2 and one 3. Their product is 6. No second 3 is shared, so  $\text{HCF}(18, 30) = 6$ . With powers, take the smaller exponent of each common prime.

## 3. Cover both lists for LCM

Use one 2, two 3s and one 5:  $2 \times 3 \times 3 \times 5 = 90$ . This contains enough prime factors to be divisible by both 18 and 30. Take the greater exponent for every prime that appears.

## 4. Check the answer and its meaning

6 divides both numbers; 90 is divisible by both. For two positive integers,  $\text{HCF} \times \text{LCM}$  equals their product:  $6 \times 90 = 18 \times 30 = 540$ . Do not assume this two-number identity works for three numbers.

## Remember

HCF is for the largest equal grouping or size. LCM is for the earliest shared repeat. The word “largest” or “smallest” alone is not enough: identify what must divide what.

## Try it and explain it

Choose an answer for each question. Use the space to show your working. A hint is available online if you need it.

1. What is  $\text{HCF}(24, 36)$ ?

A. 6 B. 12 C. 72

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2. Two bells ring now, then every 8 and 12 minutes. When will they next ring together?

A. 4 minutes B. 20 minutes C. 24 minutes

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3. For 12 and 20, the HCF is 4. What is the LCM?

A. 60 B. 80 C. 240

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### Explain in your own words

Create an equal-group story and a repeating-event story using 12 and 20. Explain why the answers are different.

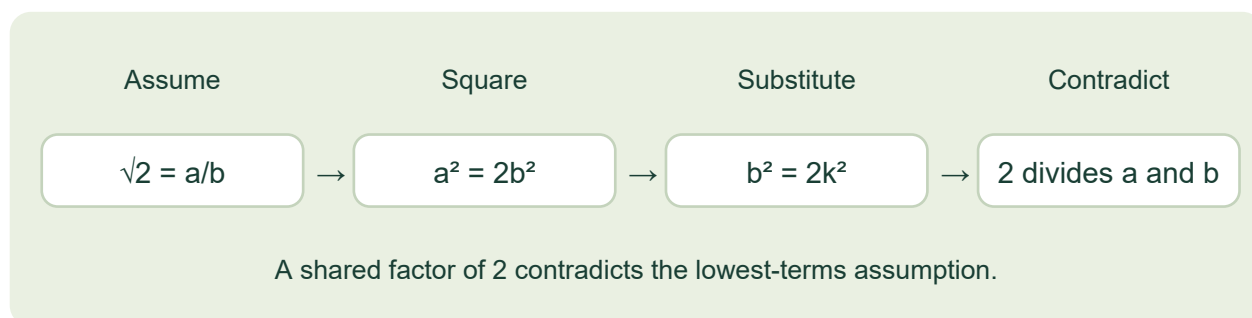
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# Explain why a root is irrational

Build a contradiction proof for  $\sqrt{2}$ ,  $\sqrt{3}$  or  $\sqrt{5}$  and explain the role of lowest terms.



## 1. Assume a fraction in lowest terms

To test  $\sqrt{2}$ , suppose  $\sqrt{2} = a/b$  with positive coprime integers  $a$  and  $b$ . Using lowest terms is essential: it gives us something precise to contradict.

## 2. Square and use prime divisibility

Squaring gives  $a^2 = 2b^2$ , so 2 divides  $a^2$ . If a prime divides a square, it divides the original integer: the prime factors of  $a^2$  are just those of  $a$  repeated. Thus 2 divides  $a$ ; write  $a = 2k$ .

## 3. Substitute to test the denominator

Now  $(2k)^2 = 2b^2$ , so  $4k^2 = 2b^2$  and  $b^2 = 2k^2$ . Therefore 2 divides  $b^2$  and hence  $b$ . Both numerator and denominator have a factor 2.

## 4. Find the contradiction

We assumed  $a$  and  $b$  were coprime, but both are divisible by 2. The assumption that  $\sqrt{2}$  is rational is impossible, so  $\sqrt{2}$  is irrational. The same argument uses the prime 3 or 5 for  $\sqrt{3}$  or  $\sqrt{5}$ .

## Remember

A calculator showing many decimal digits does not prove irrationality. Also, a square root is not always irrational:  $\sqrt{9} = 3$ . The divisibility argument uses a prime.

## Try it and explain it

Choose an answer for each question. Use the space to show your working. A hint is available online if you need it.

1. Why begin with  $a/b$  in lowest terms?

A. To make  $a$  smaller B. To ensure  $a$  and  $b$  have no shared factor greater than 1 C. To make  $b$  prime

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2. If  $a^2 = 3b^2$  and  $a = 3k$ , what follows?

A.  $b^2 = 3k^2$  B.  $b^2 = k^2$  C.  $b^2 = 9k^2$

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3. Which statement is correct?

A. Every square root is irrational B. Long calculator output proves irrationality C.  $\sqrt{5}$  is irrational, but  $\sqrt{25}$  is rational

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### Explain in your own words

Write the proof for  $\sqrt{5}$  in your own words. Identify the exact line that contradicts the starting assumption. A teacher or partner should check your reasoning.

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## Try without hints

Answer all six. Check with the separate solutions when you finish. Aim for at least 5 of 6, then revisit any missed topic.

1. Which is a prime factorisation of 150?

A.  $2 \times 3 \times 5 \times 5$  B.  $10 \times 15$  C.  $2 \times 3 \times 25$

2. Can two correct prime factorisations of the same number use different primes?

A. Yes, if the tree changes B. Yes, for even numbers C. No; only the order can change

3. What is the greatest number of identical packs from 18 pencils and 24 erasers, with none left?

A. 6 packs B. 12 packs C. 72 packs

4. Two signals repeat every 6 and 15 seconds. After a shared signal, the next shared one is after...

A. 3 seconds B. 21 seconds C. 30 seconds

5. In a proof for  $\sqrt{5}$ , which conclusion contradicts  $a/b$  being in lowest terms?

A.  $a$  is positive B. Both  $a$  and  $b$  are divisible by 5 C.  $b$  is nonzero

6. If  $r = 2 + \sqrt{5}$  were rational, then  $\sqrt{5} = r - 2$  would be...

A. rational, creating a contradiction B. zero C. an integer by definition

## Make the learning stick

One idea I can now explain:

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One mistake I corrected, and why:

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One idea I will practise next:

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### Review with a teacher or partner

Can you explain the reason for each step? Can you check a factorisation by multiplying? Can you distinguish equal-group and repeating-event problems? Can you identify exactly what contradicts lowest terms in an irrationality proof?

### Online completion

Pass all three practice checks, score at least 5 of 6 on the latest final check, and save a reflection. This records a learning activity, not a formal grade or certificate. Written proof and project work still need discussion.